Exercise 17

Verify that the solution to Equation 1 can be written in the form $x(t) = A\cos(\omega t + \delta)$.

Solution

Equation (1) is the equation of motion for a mass on a spring with no damping.

$$m\frac{d^2x}{dt^2} + kx = 0 \tag{1}$$

Find the first derivative of $x(t)$.

$$\frac{dx}{dt} = -A\omega \sin(\omega t + \delta)$$

Find the second derivative of $x(t)$.

$$\frac{d^2x}{dt^2} = -A\omega^2 \cos(\omega t + \delta)$$

Substitute these formulas into equation (1).

$$m[-A\omega^2 \cos(\omega t + \delta)] + k[A\cos(\omega t + \delta)] \stackrel{?}{=} 0$$

$$A(-m\omega^2 + k)\cos(\omega t + \delta) \stackrel{?}{=} 0$$

Since $\omega^2 = k/m$,

$$0 = 0,$$

which means $x(t) = A\cos(\omega t + \delta)$ is a solution to equation (1).